SOME GENERIC PROPERTIES OF A LOGIC MODEL FOR ANALYZING HARDWARE MAINTENANCE AND DESIGN CONCEPTS*

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SUMMARY

A mathematical structure for diagnostic logic modeling was formulated, which allows the intrinsic properties of a complex Logic Model to be studied in an abstract setting. As a result, it was found that a loop-free Logic Model is a partially ordered set and that every permutation of the elements in the terminal sec of a finite partially ordered set S partitions S into disjoint subsets. Based on these results, it was deduced that the minimum number of test points required for conclusive detection of malfunctioning components for a loop-free system is equal to the number of elements in the terminal set; this set constitutes the optimal choice for test points. Also, it was established, for each permutation of the elements in the terminal set, a relative failure probability measure. Based on this probability measure, an optimal diagnostic strategy was defined in accordance with Bellman's Principle of Optimality. Finally, for the purpose of illustration, some examples are given.

INTRODUCTION

Techniques used to analyze maintenance characteristics have been frequently subjective and difficult to determine at the conceptual and development stages of an equipment. This is largely due to the lack of analytical methods which allow the functional relationships between the hardware components or parts to be easily understood. Studies (1), (2), conducted by the US Army Air Mobility R&D Laboratory in reliability and maintainability technology, address this problem area.

The studies, entitled "Maintenance Logic Model Analysis Feasibility Investigation" and "LOGMOD Diagnostic Procedures for Mechanical and Electrical Equipment," address the feasibility of the computer generation and manipulation of a Logic Model Algorithm for use as a design and maintenance tool. Included are diagnostic procedures for assessing troubleshooting and maintenance strategies associated with hardware design.

Automatic data processing techniques are used to generate an algorithm, which accepts engineering functional design data and organizes these data into a structure of least dependent to most dependent hardware elements. The structure, called the Logic Model (Diagram), is precisely a schematic representation of the intrinsic functional relationships between the elements of the equipment. The Logic Model was developed to provide improved troubleshooting analysis and evaluation of hardware design concepts for maintenance analysis purposes.

The Logic Model concept is considered an engineering innovation. Therefore, a well established mathematical basis for this concept should be formulated. This then allows the intrinsic properties of a complex Logic Model to be studied in an abstract setting. Some preliminary results in this regard have been published (3). The purpose here is to present a mathematical development of the generic concepts.

PARTIALLY ORDERED SETS

Well established mathematical concepts form the basis upon which the techniques discussed here are based.

Generally speaking, a partially ordered set P is a system consisting of a set S and a relation satisfying the following postulates:

1. $a \sim b$ and $b \sim a$ hold if and only if $a = b$, $a, b \in S$.
2. $a \sim b$ and $b \sim c$ imply $a \sim c$, $a, b, c \in S$.

If S contains only a finite number of elements, then P is called a finite partially ordered set. In this paper, we are concerned with finite partially ordered sets. For the purpose of illustration, the following are simple examples of partially ordered sets:

A. Let S = {1, 0, -4, 10} and the relation ~ be the usual "greater than or equal to" binary relation. The system is a finite partially ordered set, for obviously the set S together with the given relation satisfies the two postulates above.

B. Let S be the set of all real numbers in the unit interval with the relation as that of example A. Then the system is a partially ordered set, but not finite.

C. Let S be the set of all letters in the word "relation" and the relation is defined by the arrangement: relation, that is, e ~ 1 (read e *precedes* 1)*;* this is a partially ordered set.

However, if we let S be the set of all lines on a plane and the relation be "parallel;" the system is *not* a partially ordered set even though it satisfies the transitive property 2 but not the asymmetric requirement I.

BASIC ELEMENTS OF THE LOGIC MODEL

The following are the basic elements with which the logic model is constructed. For a more detailed discussion the reader is referred the published reports (l) and (2).

Component - A physical component of an equipment.

Event - A measurable or observable quantity.

Functional entity - A component or an event.

Dependence - A functional relationship between two functional entities.

However, on a time basis, strategy A is optimal as shown in Table 2. This is an interesting situation. A time-cost tradeoff must be made if a single best strategy is to be chosen.

Figure 5 plots the values of expected time versus expected Lost for each strategy and indicates the overall preferred strategy to be B.

This type of analysis is normally performed by a computer for models of any significant size where the number of strategies grows factorially with the number of test points.

CONCLUSIONS

The logic modeling concept is considered to be an engineering innovation. In this paper, the mathematical structure for a class of logic models was established, and some of its generic properties, useful for maintenance analysis, were deduced. Specifically, we make the following concluding remarks:

1. A loop-free functional logic model is a partially ordered set.

2. For every finite partially ordered set, the terminal set is not empty.

3. Each permutation of the elements in the terminal set T of a finite partially ordered set S partitions S into disjoint subsets whose union is precisely S.

4. The minimum number of test points required for conclusive detection of malfunctioning components of a loop-free functional system is equal to the number of elements in the terminal set. This set constitutes the optimal choice for test points.

5. The conclusion of statement 4 is valid for systems containing loops only if the loop can be degenerated into a functional entity which

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